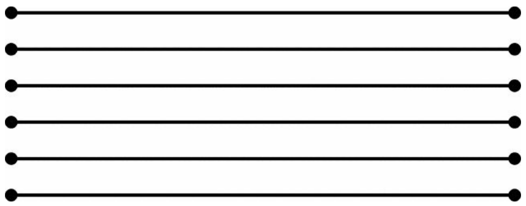
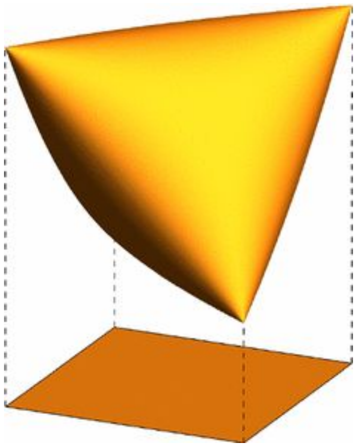
Parallel Semidefinite Optimization

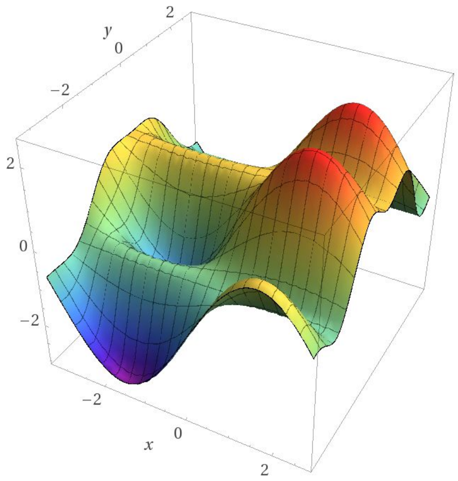
An Introduction

By: Timothy Dunn

TECHNISCHE UNIVERSITÄT DARMSTADT







TUD Graphisch-Interaktive Systeme

Optimization

Optimization

You are a Kindergarten teacher...

Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

… buying art supplies for students ...

TUD Graphisch-Interaktive Systeme

Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

… and you can't buy enough for everyone

Optimization

How do you make the most kids happy?

Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

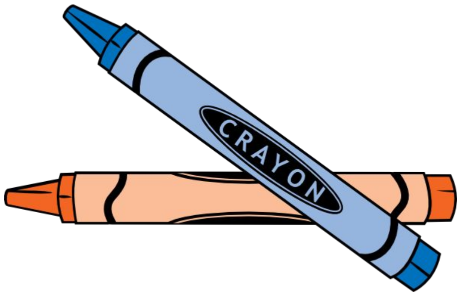
Restrictions

- You have a budget:
  - $24 to spend total

Optimization

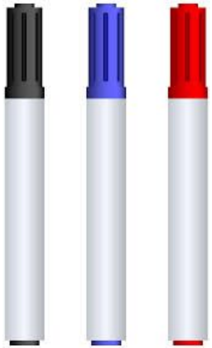
Crayons

$1



Markers

$2



Optimization

Restrictions

- You have a budget:
  - $24 to spend total
  - $1 per box of crayons
  - $2 per box of markers

Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

Restrictions

- You have a budget:
  - $24 to spend total
  - $1 per box of crayons
  - $2 per box of markers
- Students are picky:
  - 20 students want crayons
  - 10 students want markers

Restrictions

- You have a budget:
  - \$24 to spend total
  - \$1 per box of crayons
  - \$2 per box of markers
- Students are picky:
  - 20 students want crayons
  - 10 students want markers
- You want to be fair
  - Can’t buy more than 3 more boxes of crayons than markers

Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

How do we optimize?

Optimization

How do we optimize?

let x_1 denote boxes of crayons

let x_2 denote boxes of markers

maximize $x_1 + x_2$

Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

Restrictions

- You have a budget:
    - $24 to spend total
    - $1 per box of crayons
    - $2 per box of markers
- Students are picky:
    - 20 students want crayons
    - 10 students want markers
- You want to be fair
    - Can't buy more than 3 more boxes of crayons than markers

TUD Graphisch-Interaktive Systeme

Optimization

Restrictions

- You have a budget:
  - $24 to spend total
  - $1 per box of crayons
  - $2 per box of markers
- Students are picky:
  - 20 students want crayons
  - 10 students want markers
- You want to be fair
  - Can’t buy more than 3 more boxes of crayons than markers

$$x_1 + 2x_2 \leq 24$$

Optimization

Restrictions

- You have a budget:
  - $24 to spend total
  - $1 per box of crayons
  - $2 per box of markers
- Students are picky:
  - 20 students want crayons
  - 10 students want markers
- You want to be fair
  - Can’t buy more than 3 more boxes of crayons than markers

$$x_1 + 2x_2 \leq 24$$

$$x_1 \leq 20 \qquad x_2 \leq 10$$

Optimization

Restrictions

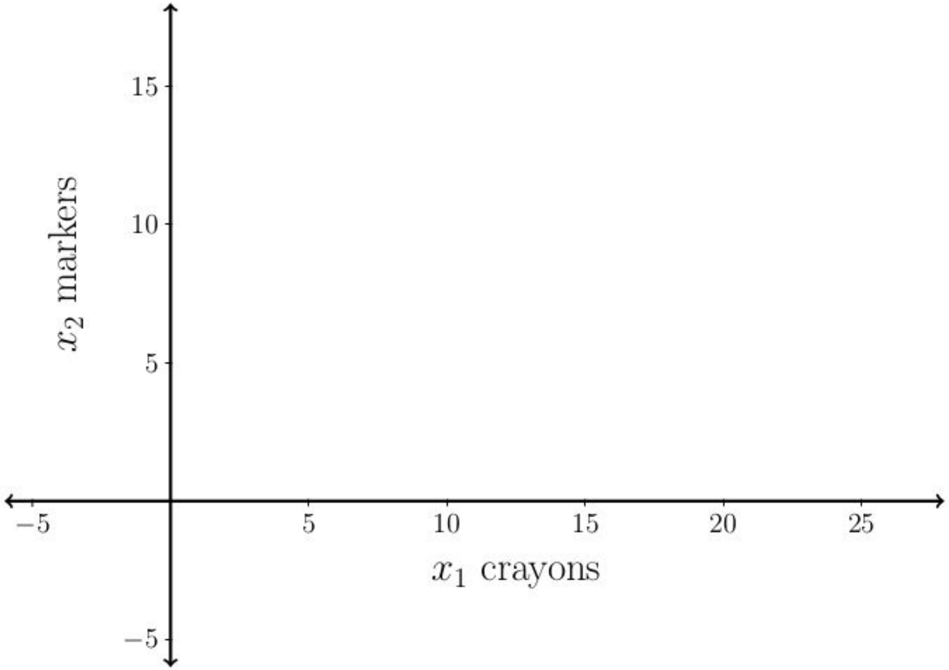
- You have a budget:
  - $24 to spend total
  - $1 per box of crayons
  - $2 per box of markers
- Students are picky:
  - 20 students want crayons
  - 10 students want markers
- You want to be fair
  - Can't buy more than 3 more boxes of crayons than markers

$$x_1 + 2x_2 \leq 24$$

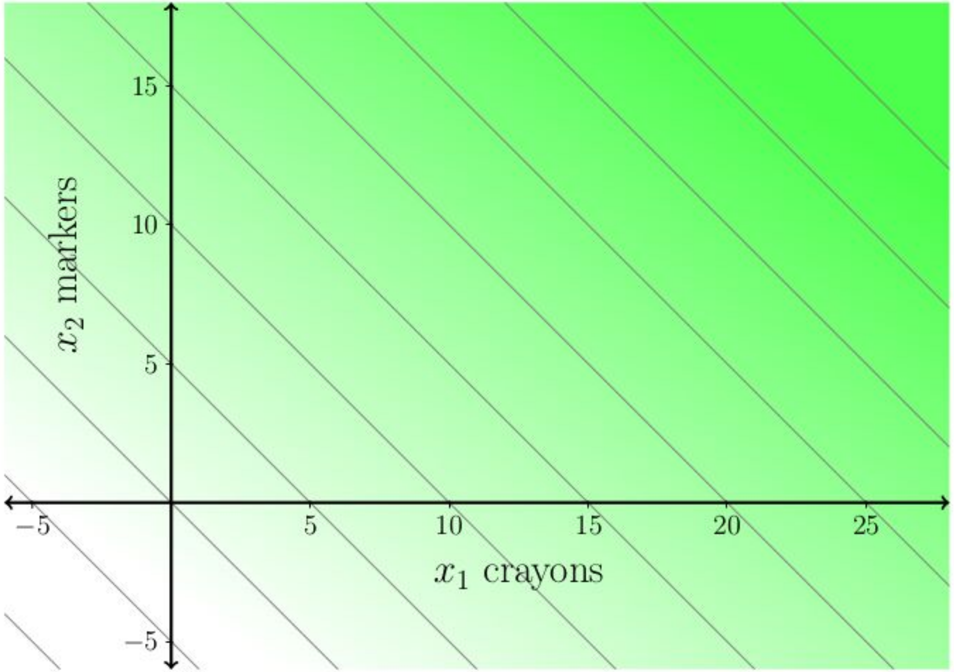
$$x_1 \leq 20 \qquad x_2 \leq 10$$

$$x_1 - x_2 \leq 3$$

Optimization



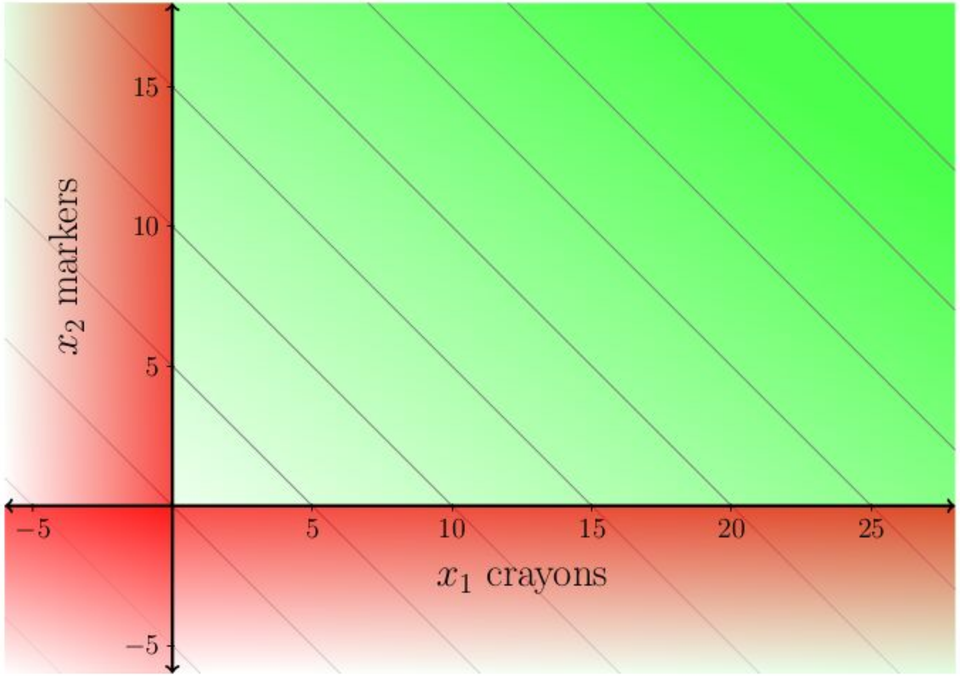
Optimization



maximize

$$x_1 + x_2$$

Optimization



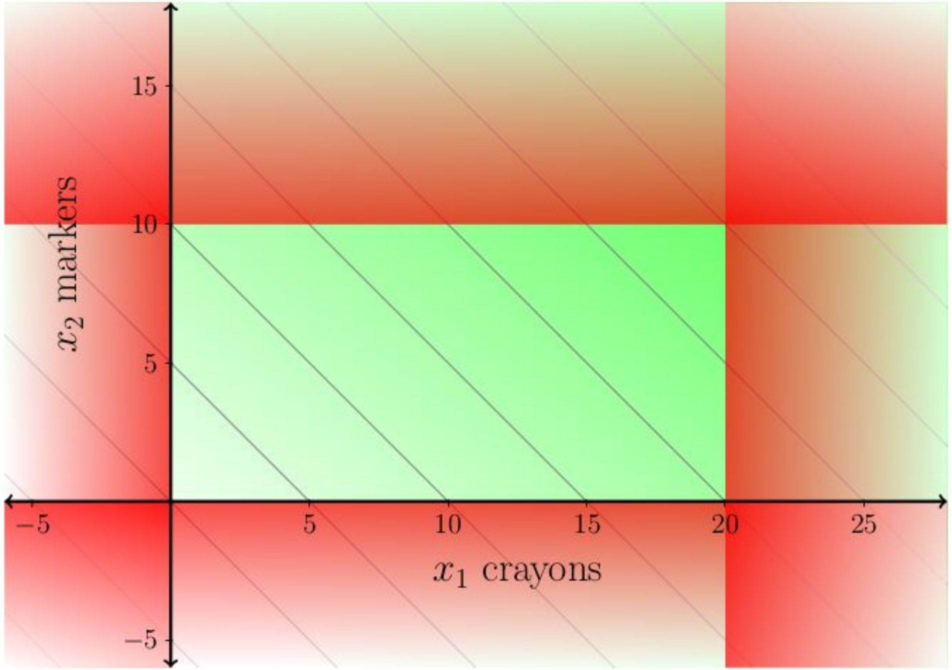
maximize

$$x_1 + x_2$$

such that

$$x_1, x_2 \geq 0$$

Optimization



maximize

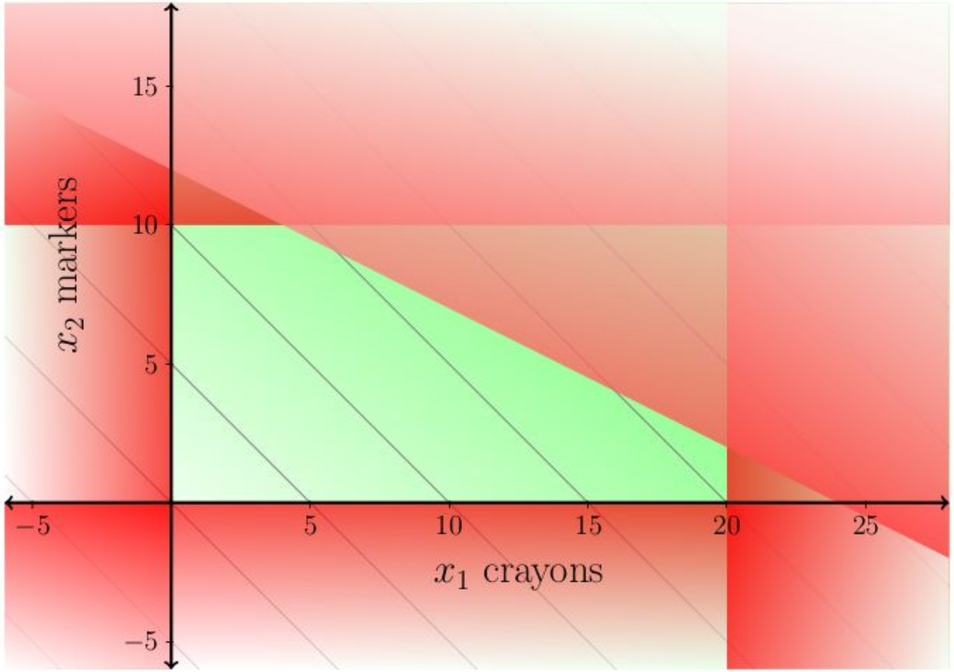
$$x_1 + x_2$$

such that

$$x_1, x_2 \geq 0$$

$$x_1 \leq 20 \qquad x_2 \leq 10$$

Optimization



maximize

$$x_1 + x_2$$

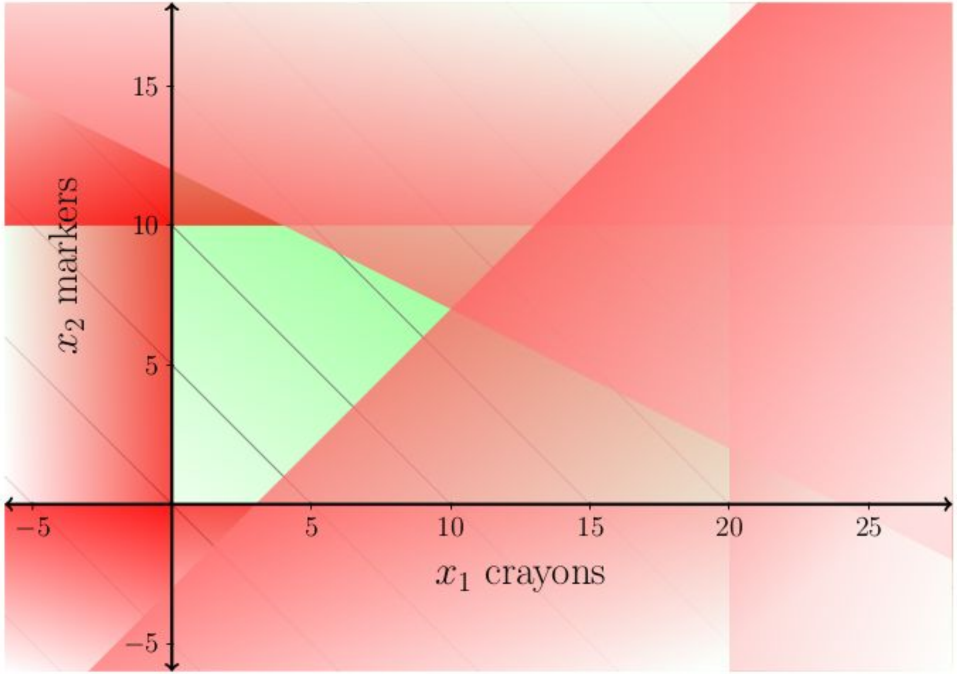
such that

$$x_1, x_2 \geq 0$$

$$x_1 \leq 20 \qquad x_2 \leq 10$$

$$x_1 + 2x_2 \leq 24$$

Optimization



maximize

$$x_1 + x_2$$

such that

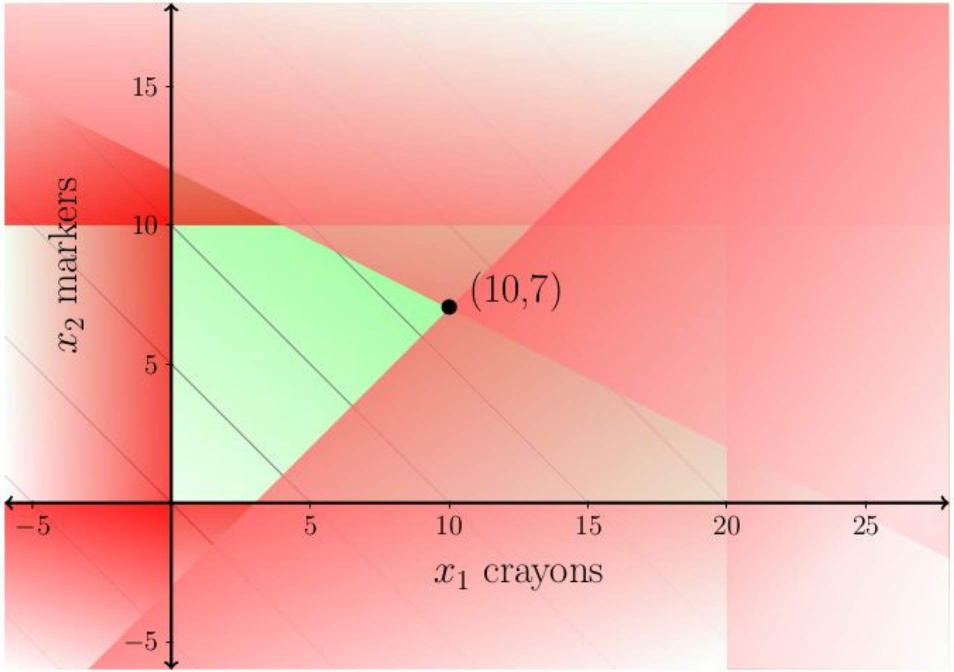
$$x_1, x_2 \geq 0$$

$$x_1 \leq 20 \qquad x_2 \leq 10$$

$$x_1 + 2x_2 \leq 24$$

$$x_1 - x_2 \leq 3$$

Optimization



maximize

$$x_1 + x_2$$

such that

$$x_1, x_2 \geq 0$$

$$x_1 \leq 20 \qquad x_2 \leq 10$$

$$x_1 + 2x_2 \leq 24$$

$$x_1 - x_2 \leq 3$$

Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

LP Formal Definition

maximize

$$c_1x_1 + c_2x_2 + ... + c_nx_n$$

such that

$$x_1, x_2, ..., x_n \geq 0$$

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$$

maximize

$$x_1 + x_2$$

such that

$$x_1, x_2 \geq 0$$

$$x_1 \leq 20 \qquad x_2 \leq 10$$

$$x_1 + 2x_2 \leq 24$$

$$x_1 - x_2 \leq 3$$

TECHNISCHE UNIVERSITÄT DARMSTADT

LP Formal Definition

maximize

$$c_1x_1 + c_2x_2 + ... + c_nx_n$$

such that

$$x_1, x_2, ..., x_n \geq 0$$

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$$

maximize

$$\langle c^T, x \rangle$$

such that

$$x \geq 0$$

$$Ax \leq b$$

TUD Graphisch-Interaktive Systeme

Linear Programs

TECHNISCHE UNIVERSITÄT DARMSTADT

Primal	Dual
maximize	minimize
$\langle c^T, x\rangle$	$\langle b^T, y\rangle$
such that	such that
$x \geq 0$	$y \geq 0$
$Ax \leq b$	$A^T y \geq c$

TUD Graphisch-Interaktive Systeme

Semidefinite Programs

TECHNISCHE UNIVERSITÄT DARMSTADT

LP Primal	**SDP Primal**
maximize	maximize
$\langle c^T, x \rangle$	$\langle C, X \rangle$
such that	such that
$x \geq 0$	$X \succeq 0$
$Ax \leq b$	$\langle A_k, X \rangle \leq b_k \quad \text{for } k = 1...m$

TUD Graphisch-Interaktive Systeme

Semidefinite Programs

$$X \succeq 0$$

Semidefinite Programs

$$X \succeq 0$$

$$\forall v \in \mathbb{R}^n, \quad v^T X v \geq 0$$

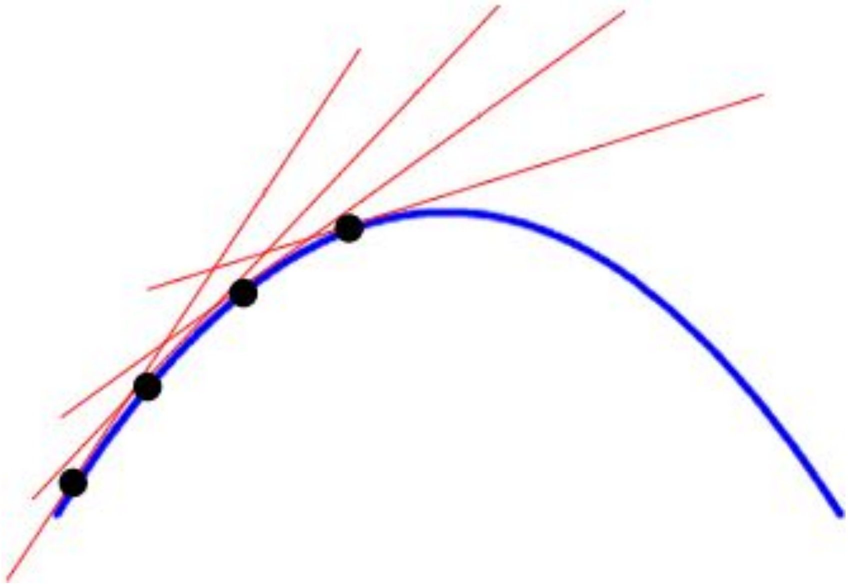
Semidefinite Programs

$$X \succeq 0$$

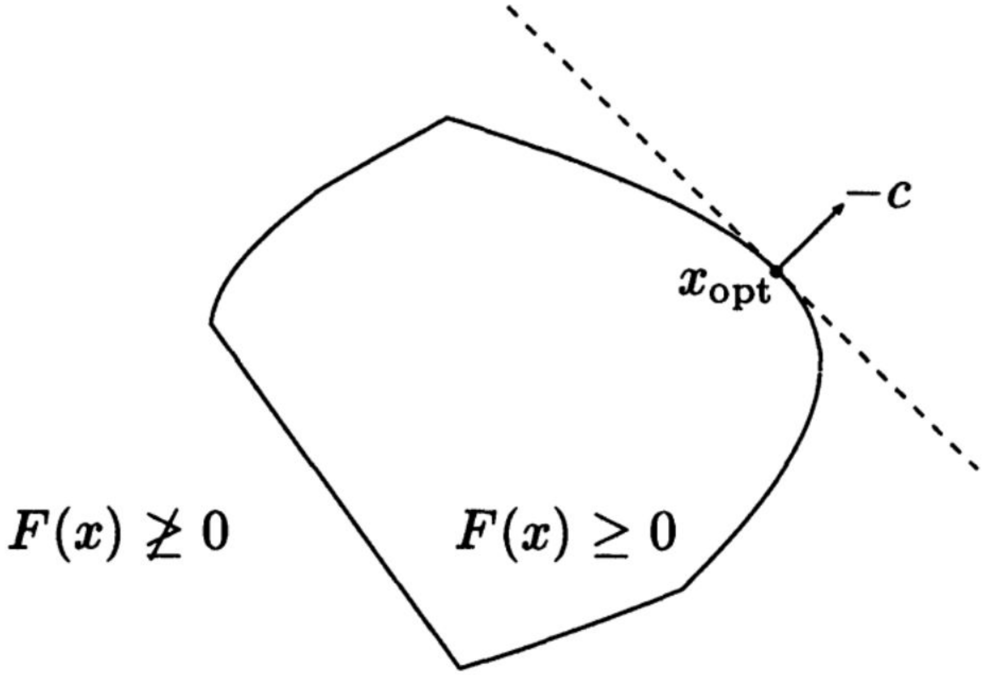
$$\forall v \in \mathbb{R}^n, \quad v^T X v \geq 0$$

This is equivalent to
an infinite set of linear constraints!

Semidefinite Programs

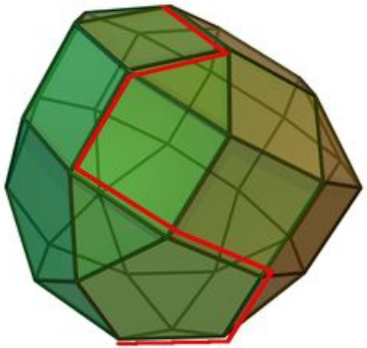


Semidefinite Programs

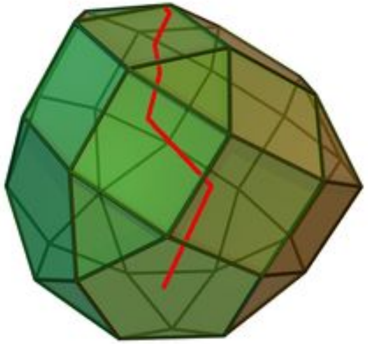


Semidefinite Programs

Simplex Method

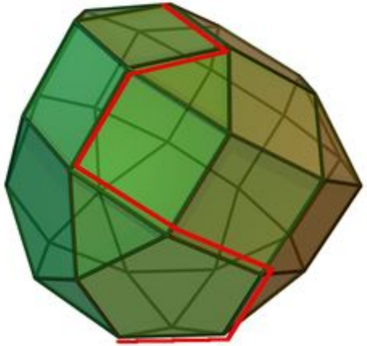


Interior Point Methods

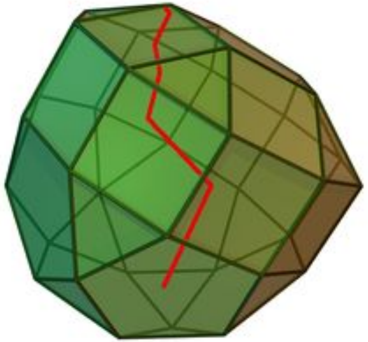


Semidefinite Programs

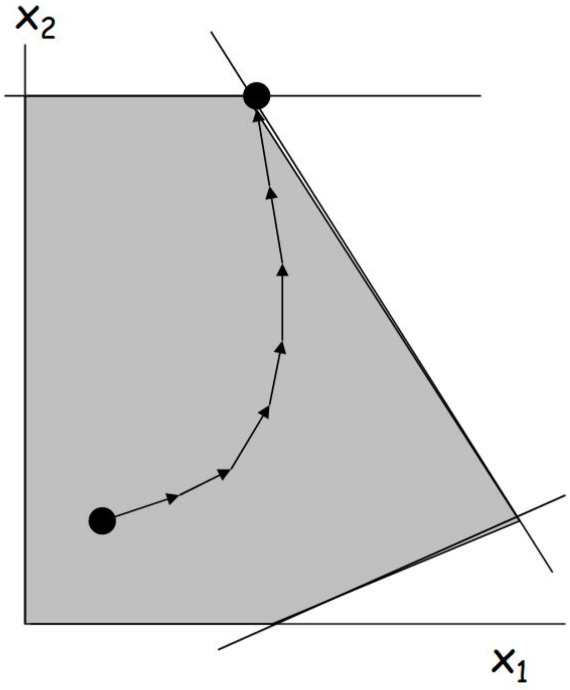
~~Simplex Method~~



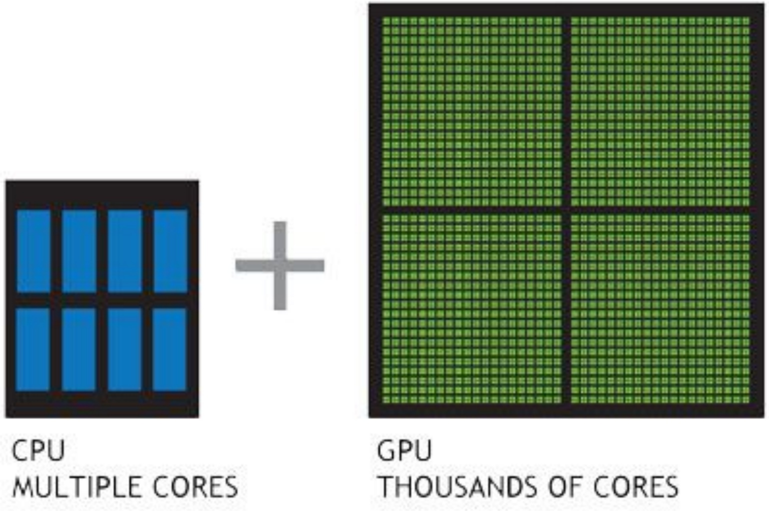
Interior Point Methods



Semidefinite Programs



Parallel Programming



Parallel Programming

TECHNISCHE UNIVERSITÄT DARMSTADT

SDP Primal

$$C, X, A_k \in \mathbb{R}^{n \times n}$$

maximize

$$\langle C, X \rangle$$

such that

$$X \succeq 0$$

$$\langle A_k, X \rangle \leq b_k \quad \text{for } k = 1 \ldots m$$

TUD Graphisch-Interaktive Systeme

Parallel Programming: Saving Space

SDP Primal

maximize

$$\langle C, X \rangle$$

such that

$$X \succeq 0$$

$$\langle A_k, X \rangle \leq b_k \quad \text{for } k = 1...m$$

$$C, X, A_k \in \mathbb{R}^{n \times n}$$

$$\text{let } n = 600$$

$$\text{let } m = 15$$

Parallel Programming: Saving Space

SDP Primal

maximize

$$\langle C, X \rangle$$

such that

$$X \succeq 0$$

$$\langle A_k, X \rangle \leq b_k \quad \text{for } k = 1 \ldots m$$

$$C, X, A_k \in \mathbb{R}^{n \times n}$$

$$\text{let } n = 600$$

$$\text{let } m = 15$$

$$6 \text{ MB}$$

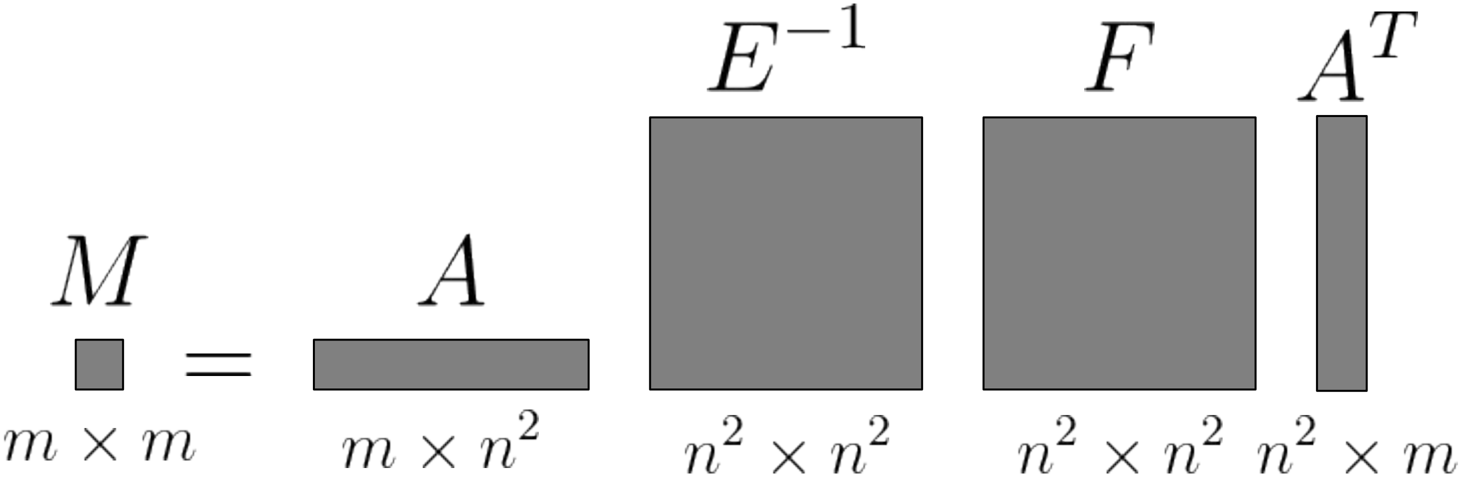
Parallel Programming: Saving Space

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ E & 0 & F \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} b - Ax \\ c - z - A^T y \\ \text{svec}(\sigma\mu I - H_P(XZ)) \end{bmatrix}$$

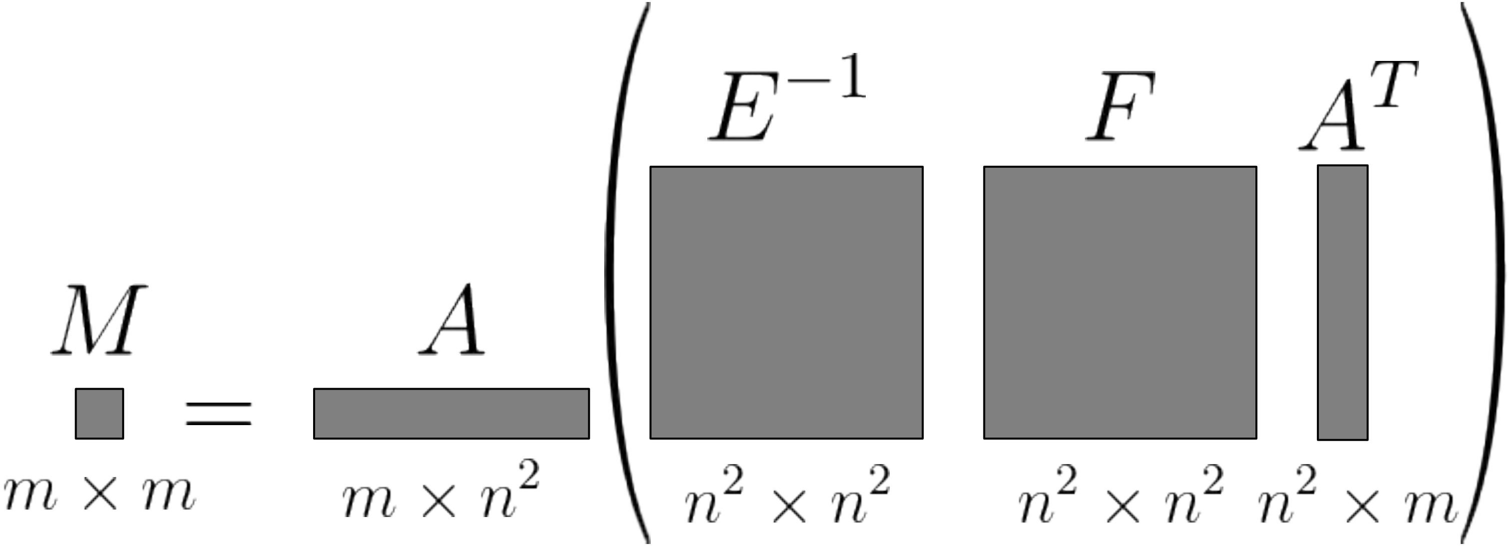
$$M\delta y = h$$

$$M = AE^{-1}FA^T$$

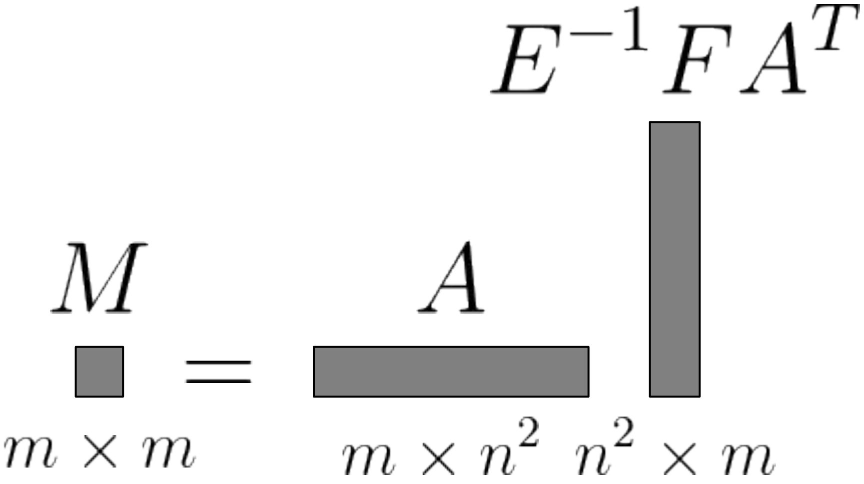
Parallel Programming: Saving Space



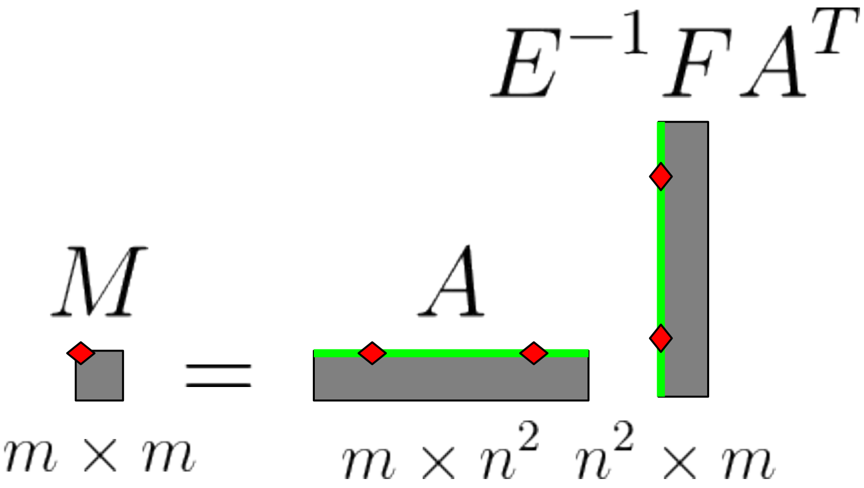
Parallel Programming: Saving Space



Parallel Programming: Saving Space

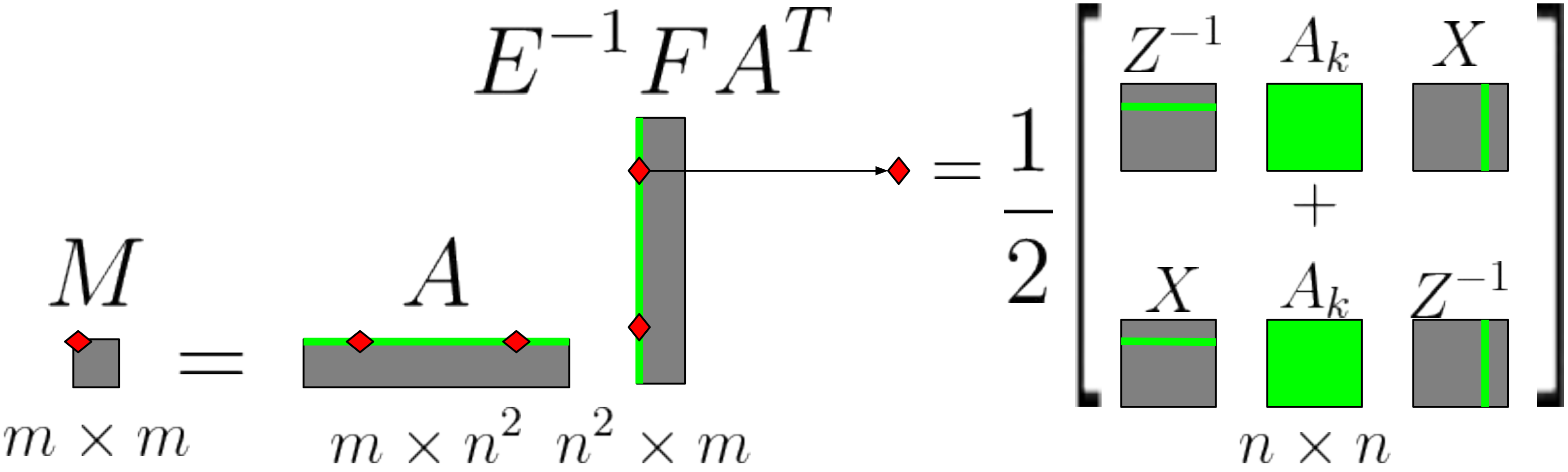


Parallel Programming: Saving Space



Parallel Programming: Saving Space

TECHNISCHE UNIVERSITÄT DARMSTADT



Questions?

Acknowledgements

Daniel Thuerck

DAAD Rise Program

Clarkson Honors Program

Images

[1] https://drawinglics.com/photos/2090385/children-holding-hands-clipart-10113-kids-holding-hands-vector-art-illustration.py
[2] https://clipartpng.com/?1368,markers-png-clip-art
[3] https://link.springer.com/article/10.1007/s10107-015-0922-1
[4] https://www.matroid.com/blog/post/the-hard-thing-about-deep-learning
[5] https://en.wikipedia.org/wiki/Simplex_algorithm
[6] http://www.stat.cmu.edu/~ryantibs/convexopt-F13/lectures/18-semidefiniteprogramming.pdf
[7] http://slideplayer.com/slide/8074853/
[8] https://people.duke.edu/~ccc14/sta-663/CUDAPython.html